$$= \sqrt{\frac{D_{2A}}{D_{1A}}}\,\xi_{1A} = \frac{y}{y'}\,\xi'_{2A} = \frac{y}{y'}\sqrt{\frac{D_{2A}}{D_{2B}}}\,\xi'_{2B} \quad (24)$$

The constants in Equations (21) to (23) can be expressed as

$$A_1 = \frac{H\, C_{1A,0}\sqrt{\frac{D_{1A}}{D_{2A}}}\,\mathrm{erf}\,\xi'_{2A}}{1 + H\sqrt{\frac{D_{1A}}{D_{2A}}}\,\mathrm{erf}\,\xi'_{2A}},$$

$$B_1 = \frac{-C_{1A,0}}{1 + H\sqrt{\frac{D_{1A}}{D_{2A}}}\,\mathrm{erf}\,\xi'_{2A}}, \quad A_2' = \frac{-q\,\mathrm{erf}\,\xi'_{2B}}{\mathrm{erfc}\,\xi'_{2B}}$$

$$A_2 = \frac{C_{1A,0}\sqrt{\frac{D_{1A}}{D_{2A}}}\,\mathrm{erf}\,\xi'_{2A}}{1 + H\sqrt{\frac{D_{1A}}{D_{2A}}}\,\mathrm{erf}\,\xi'_{2A}},$$

$$B_2 = \frac{-C_{1A,0}\sqrt{\frac{D_{1A}}{D_{2A}}}}{1 + H\sqrt{\frac{D_{1A}}{D_{2A}}}\,\mathrm{erf}\,\xi'_{2A}}, \quad B_2' = \frac{q}{\mathrm{erfc}\,\xi'_{2B}} \quad (25)$$

The time dependence of location of reaction zone y' can be shown to be expressed as*

$$y' = 2\alpha_1 \sqrt{\int_0^t S^2(x)\,dx} \Big/ S(t) \quad (26)$$

where constant α_1 is obtained from the solution of the equation

$$\sqrt{D_{2A}}\,B_2\,e^{-(\alpha_1^2/D_{2A})} + \sqrt{D_{2B}}\,B_2'\,e^{-(\alpha_1^2/D_{2B})} = 0 \quad (27)$$

* Supplementary material has been deposited as Document No. 01928 with the National Auxiliary Publications Service (NAPS), c/o CCM Information Corp., 866 Third Ave., New York 10022 and may be obtained for \$2.00 microfiche and \$5.00 for photocopies.

The mass transfer rate for this case is

$$N = -D_{2A}\,B_2\,f_{2A}(t) \quad (28)$$

and the enhancement factor Φ can be shown to be

$$\Phi = 1/\mathrm{erf}\,(\alpha_1/\sqrt{D_{2A}}) \quad (29)$$

a value independent of time-dependence of interfacial area.

NOTATION

C_i = concentration of transferring species in ith phase
D_i = diffusion coefficient of transferring species in ith phase
H = equilibrium dissolution constant
K = intrinsic reaction rate constant
q = initial concentration of species B in the liquid phase
t = time
y = distance normal to fluid-liquid interface

Subscripts

i refers to phase $i = 1$ fluid; $i = 2$ liquid
A, B, refers to the transferring species A and B respectively
0 refers to condition at time $t = 0$

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Notes on Transfer in Turbulent Pipe Flow

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A recent paper by Hughmark (1971) proposes models for heat and mass transfer for the wall region and the core of turbulent pipe flow. The core model includes an empirical correlation for transfer by eddy diffusion that is a function of the Reynolds number. The data shown by Figure 6 of the paper can also be represented as a function of the friction factor by the equation

$$k^+_{EC} = 2\sqrt{f/2} \quad (1)$$

which represents the heat transfer, mass transfer, momentum relationship for the fully turbulent core region

$$\frac{h_{EC}}{\rho C_p} = k_{EC} = U\,f \quad (2)$$

It is interesting to observe that Equation (2) does not include molecular diffusion properties.

The wall region analysis includes a correlation for eddy diffusion transfer that is based upon the pipe solution mass transfer data of Harriott and Hamilton (1965). The Mizushino et al. (1971) analysis of wall region data indicates that the pipe solution method may give high mass transfer coefficients because of surface roughness associated with the dissolving pipe wall. The authors report mass transfer data with reduction of ferricyanide ions at a nickel cathode in the presence of a large excess of sodium and potassium hydroxide. Shaw and Hanratty (1964), and Hubbard and Lightfoot (1966), also report data for ferricyanide reduction. Lin et al. (1951) used a number

Table 1. Wall Region Mass Transfer Data

Investigator	Method	Average a
Shaw and Hanratty	Ferricyanide reduction	0.063
Hubbard and Lightfoot	Ferricyanide reduction	0.055
Mizushina et al.	Ferricyanide reduction	0.062
Lin et al.	Electrochemical	0.067
Meyerink and Friedlander	Wall solution	0.070
Harriott and Hamilton	Wall solution	0.078

of electrochemical reactions to obtain wall region mass transfer data. Table 1 summarizes these data to show the coefficient for the equation

$$k^+_{EW} = a\, N_{Sc}^{-2/3} \tag{3}$$

The Meyerink and Friedlander (1962) data reported in the table are for a benzoic acid pipe. The Shaw and Hanratty data are the corrected data reported by Son and Hanratty (1967). Thus Table 1 indicates that $a \approx 0.065$ is more consistent with these data than the value of $a = 0.0816$ used in the prior paper.

NOTATION

C_p = specific heat
f = Fanning friction factor
h = heat transfer coefficient
k = mass transfer coefficient
N_{Sc} = Schmidt number
U = mean velocity

Greek Letters

ρ = fluid density

Subscripts

EC = eddy diffusion, core
EW = eddy diffusion, wall

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Parameter Estimation from Transient Rate Data

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The results of transient experiments on the decomposition of nitrous oxide over nickel oxide have been reported by Yang et al. (1972), and details of gradientless catalytic reactor which was used are also available (Bennett et al., 1972). In this work a carrier gas of argon through the reactor was suddenly replaced by various mixtures of argon and nitrous oxide at the same flow rate, pressure, and temperature. The response of the well-mixed reactor to these signals was obtained by an on-line mass spectrometer as the composition changed to a new steady state. Mass and heat transfer falsification effects were absent (Bennett et al., 1972; Yang et al., 1972).

The ordinary differential equations and the algebraic constraints which describe the transient reacting system have been given by Bennett (1967), and the latest paper (Yang et al., 1972) gives the results of a search for the parameter in a sequence of steps which describes the N_2O reaction. In these previous publications no details of the method of parameter estimation were given.

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